ME 425

MECHGANICAL VIBRATIONS

Regular and Modified Tuned Mass Damper Design for Mass-Spring Chains

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1. Introduction

The system provided below in Figure 1, a modified tuned mass damper is attached to an undamped n-degrees-of-freedom mass-spring chains with masses m_i and stifnesses k_i. The system is has n+1 degrees of freedom and the modified tuned mass damper can be placed in any consecutive masses. Assuming k_b is zero, the tuned mass damper becomes a regular tuned mass damper and this assumption is utilized in the design process of regular tuned mass damper.

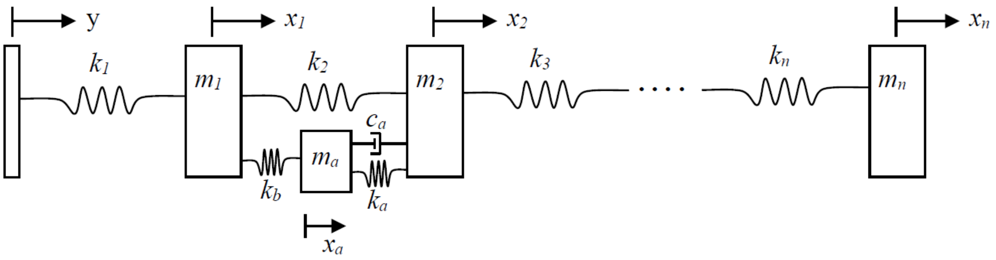

Figure 1: Schematic of the System with Tuned Mass Dampers

In this report, regular and modified tuned mass dampers are designed for the given system. Through the design process, following steps are carried out: Natural frequencies and mode shapes are calculated for the selected number of consecutive masses. Absolute transmissibility is plotted in the presence of base excitation. The system is optimized to minimize the peak transmissibility both assuming k_b is zero and vice versa, and optimized ki and ci values are provided.

Design criteria are listed below:

- $m_i=2(n-i+1)/(n^2+n)$
- $k_i=50(n^2+n)/(n-i+1)$
- n = 2, 3, 4, 5
- $0.1 < m_a < 0.3$
- $0 \leq \omega \leq 1.5\omega_n$

In this report, calculations are carried out and plots are generated using MATLAB.

1. Natural Frequencies and Mode Shapes

Assuming the tuned mass damper is not attached to the system, the free-body diagram of the resulting mass-spring chains is provided below in Figure 2.

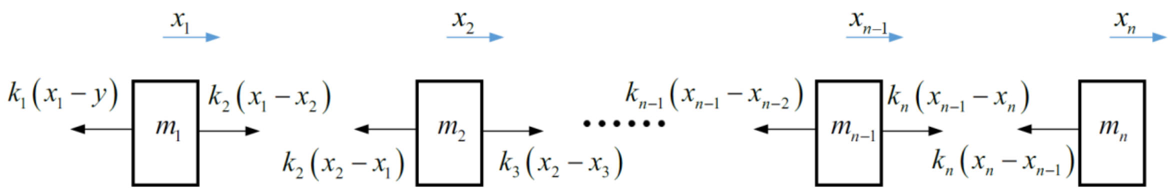

Figure 2: FBD of n-degrees-of-freedom Mass-Spring Chains without a Tuned Mass Damper

The equation of motion for the above free-body diagram in matrix form is given as

$$\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & . & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{n-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_n \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \cdot \\ \cdot \\ \cdot \\ \ddot{x}_{n-1} \\ \ddot{x}_n \end{bmatrix} + \begin{bmatrix} (k_1+k_2) & -k_2 & 0 & 0 & 0 & 0 & 0 \\ -k_2 & (k_2+k_3) & -k_3 & 0 & 0 & 0 & 0 \\ 0 & -k_3 & \cdot & . & 0 & 0 & 0 \\ 0 & 0 & . & . & . & 0 & 0 \\ 0 & 0 & 0 & . & . & . & 0 \\ 0 & 0 & 0 & 0 & k_{n-1} & (k_{n-1}+k_n) & -k_n \\ 0 & 0 & 0 & 0 & 0 & -k_n & k_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} k_1 y(t) \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \end{bmatrix}$$

$$[M]\{\ddot{x}\}+[K]\{x\}=\{F\}$$

where m_i and k_i are equal to

$$m_i = \frac{2(n-i+1)}{(n^2+n)}$$

$$k_i = \frac{50(n^2+n)}{(n-i+1)}$$

The user input, *n*, is selected as *3* and the resulting natural frequencies and mode shapes are provided in Table 1.

Table 1: Natural Frequencies and Mode Shapes

	Natural Frequency (rad/sec)	First Mode	Second Mode	Third Mode
1	12.876	1.0000	1.3904	1.4575
2	36.769	1.0000	-0.5865	-.09393
3	76.041	1.0000	-7.9705	13.1485

In Figure 3, mode shapes are scattered as deviations from the mean position.

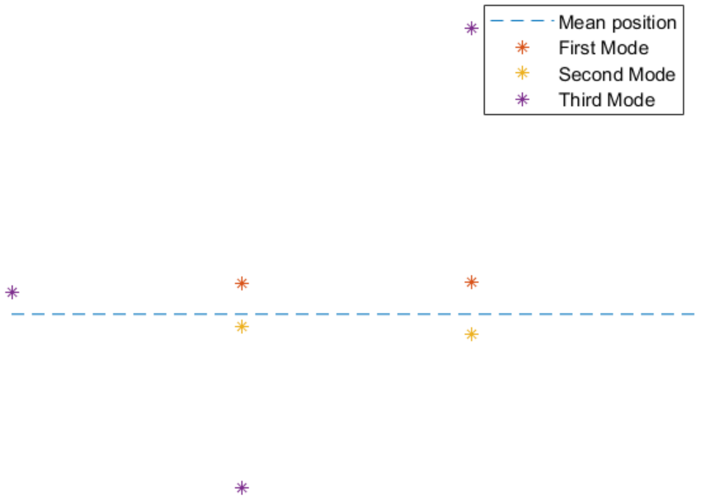

Figure 3: Mode Shapes

2. Absolute Transmissibility

The n-degrees-of-freedom system is to be base excited. Assuming $\vec{x}(t) = Xe^{i\omega t}$, $y(t) = Ye^{i\omega t}$ and $Y = 1$, transmissibility for the last mass *(Xn/Y)* is found and the absolute value of the transmissibility is plotted in log-log scale for $0 \leq \omega \leq 1.5\omega_n$ where ω_n is the largest natural frequency.

The calculations are carried out as following

$$[M]\{\ddot{x}\}+[K]\{x\}=\{F\}$$

$$\vec{x}(t)=\vec{X}e^{iot}$$

$$\ddot{\vec{x}}(t)=\vec{X}(-\omega^2)e^{i\omega t}$$

$$[M]\vec{X}(-\omega^2)e^{iot}+[K]\vec{X}e^{i\omega t}=\{F_0\}e^{i\omega t}$$

$$\{F\}=\{F_0\}e^{i\omega t}=\begin{bmatrix} k_1 \\ 0 \\ . \\ . \\ . \\ 0 \\ 0 \end{bmatrix} e^{i\omega t}$$

$$[M]\vec{X}(-\omega^2)+[K]\vec{X}=\{F_0\}$$

$$(-\omega^2[M]+[K])\vec{X}=\{F_0\}$$

and the governing equation for the transmissibility is found to be

$$\vec{X} = \frac{\{F_0\}}{(-\omega^2[M]+[K])} = \frac{b}{A}$$

The maximum transmissibility is found to be *312.698* and the corresponding plot is provided in Figure 4.

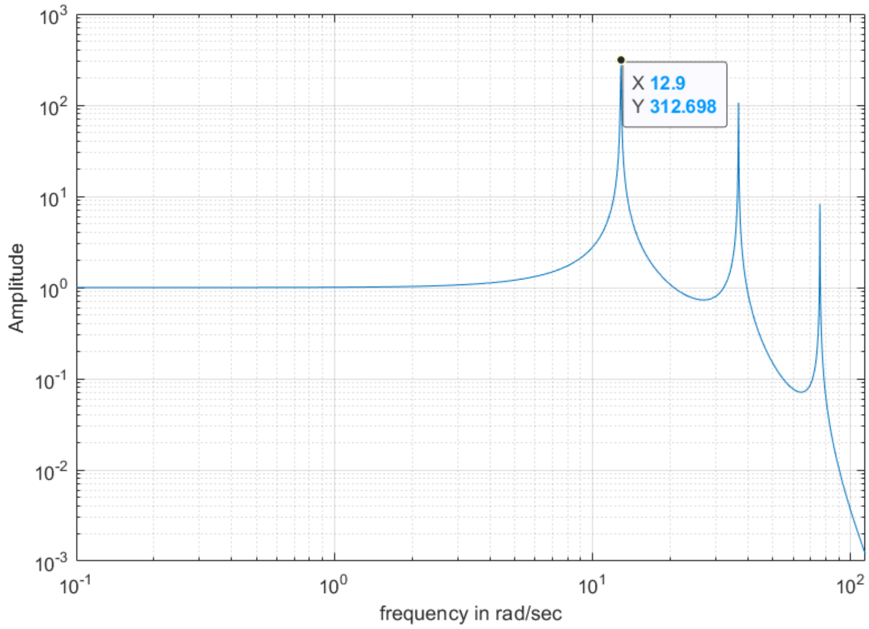

Figure 4: Transmissibility of the System without a Tuned Mass Damper

3. Regular Tuned Mass Damper Design

Assuming k_b is equal to zero, the tuned mass damper is simplifed to a regular tuned mass damper. Hence, the free-body diagram of the resulting mass-spring chains with a regular tuned mass damper is provided below in Figure 5.

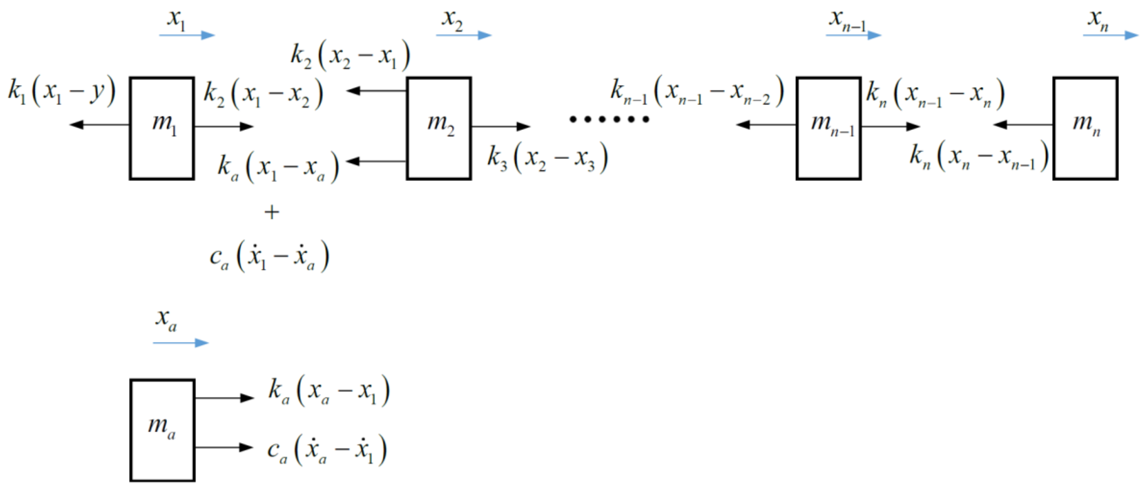

Figure 5: FBD of Mass-Spring Chains with a Regular Tuned Mass Damper

The equation of motion for the above free-body diagram in matrix form is given as

$$\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & . & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{n-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_n & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_a \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \cdot \\ \cdot \\ \ddot{x}_{n-1} \\ \ddot{x}_n \\ \ddot{x}_a \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_a & 0 & 0 & 0 & 0 & 0 & -c_a \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & . & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -c_a & 0 & 0 & 0 & 0 & 0 & c_a \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \cdot \\ \cdot \\ \dot{x}_{n-1} \\ \dot{x}_n \\ \dot{x}_a \end{bmatrix}$$

$$+ \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_2 & k_2 + k_3 + k_a & -k_3 & 0 & 0 & 0 & 0 & -k_a \\ 0 & -k_3 & k_3 + k_4 & . & 0 & 0 & 0 & 0 \\ 0 & 0 & . & . & . & 0 & 0 & 0 \\ 0 & 0 & 0 & . & . & . & 0 & 0 \\ 0 & 0 & 0 & 0 & . & k_{n-1} + k_n & -k_n & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_n & k_n & . \\ 0 & -k_a & 0 & 0 & 0 & 0 & . & k_a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ x_{n-1} \\ x_n \\ x_a \end{bmatrix} = \begin{bmatrix} k_1 y(t) \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[M_1]\{\ddot{x}\}+[C_1]\{\dot{x}\}+[K_1]\{x\}=\{F_1\}$$

The calculations are carried out as following by assuming steady-state

$$\vec{x}(t)=\vec{X}e^{i\omega t}$$

$$\ddot{\vec{x}}(t)=\vec{X}(-\omega^2)e^{i\omega t}$$

$$\{F\}=\{F_0\}e^{i\omega t}=\begin{bmatrix} k_1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \end{bmatrix} e^{i\omega t}$$

$$[M_1]\vec{X}(-\omega^2)+[C_1](i\omega)\vec{X}+[K_1]\vec{X}=\{F_1\}$$

$$(-\omega^2[M_1]+(i\omega)[C_1]+[K_1])\vec{X}=\{F_0\}$$

and the governing equation for the transmissibility is found to be

$$\vec{X}=\frac{\{F_0\}}{(-\omega^2[M_1]+(i\omega)[C_1]+[K_1])}=\frac{b_1}{A_1}$$

The mass value of tuned mass damper, m_a, is selected as 0.*3* and the optimum value of k_a and c_a are found to be *25.447* and *2.008* respectively for minimum transmissibility, which is decreased to *2.723*. During the optimization process, it is also found that the tuned mass damper is to be connected between m_1 and m_2 to achieve the minimum transmissibility. The corresponding transmissibility graph is plotted in log-log scale and provided in Figure 6.

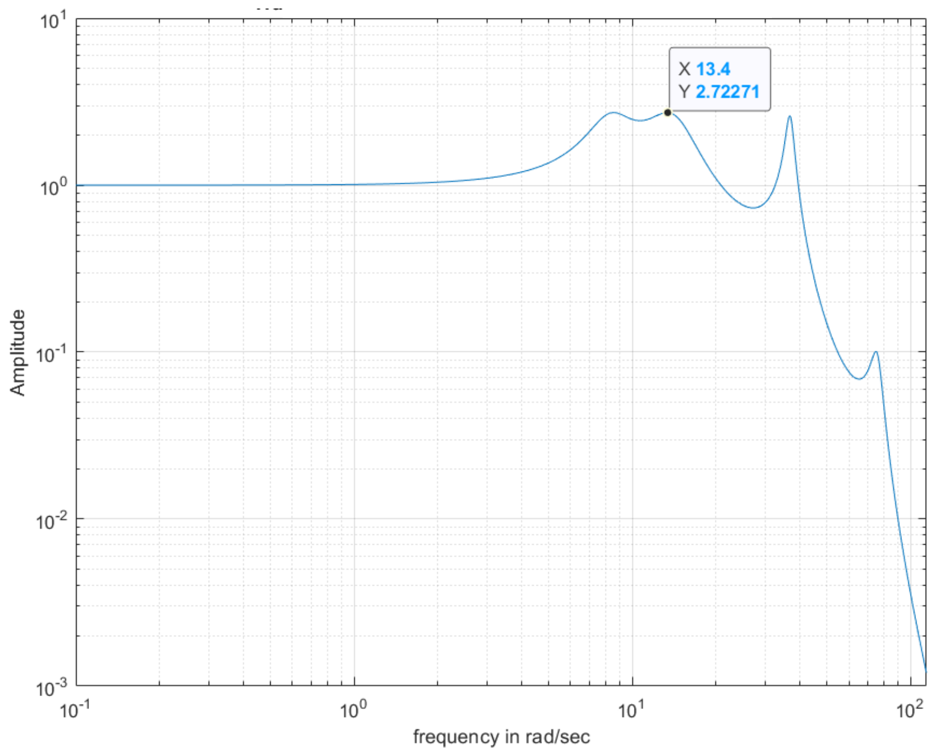

Figure 6: Transmissibility of the System with a Regular Tuned Mass Damper

4. Modified Tuned Mass Damper Design

The free-body diagram of the mass-spring chains with a regular tuned mass damper is provided below in Figure 7.

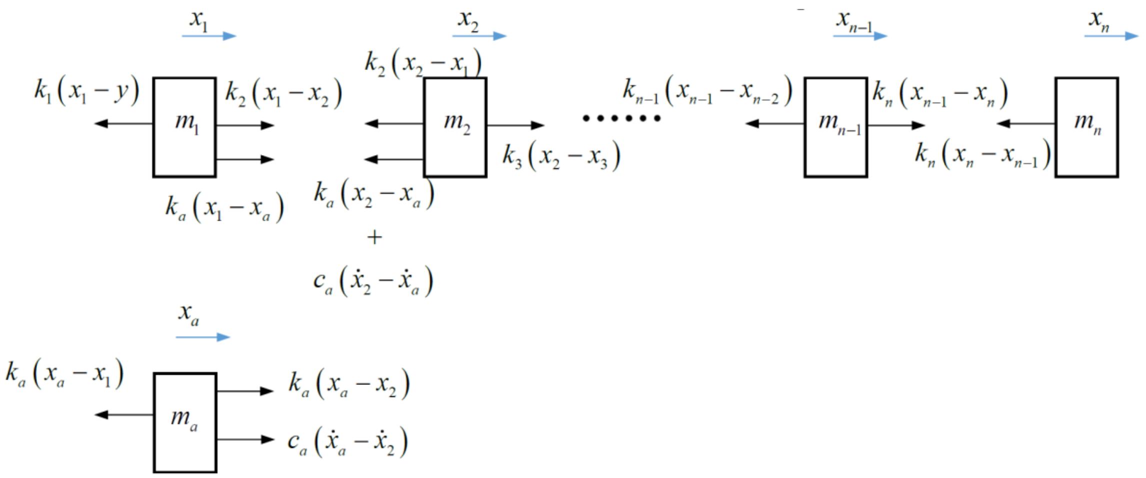

Figure 7: FBD of Mass-Spring Chains with a Modified Tuned Mass Damper

The equation of motion for the above free-body diagram in matrix form is given as

$$\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & . & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{n-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_n & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_a \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \cdot \\ \cdot \\ \ddot{x}_{n-1} \\ \ddot{x}_n \\ \ddot{x}_a \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_a & 0 & 0 & 0 & 0 & 0 & -c_a \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & . & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -c_a & 0 & 0 & 0 & 0 & 0 & c_a \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \cdot \\ \cdot \\ \dot{x}_{n-1} \\ \dot{x}_n \\ \dot{x}_a \end{bmatrix}$$

$$+ \begin{bmatrix} k_1 + k_2 + k_a & -k_2 & 0 & 0 & 0 & 0 & 0 & -k_a \\ -k_2 & k_2 + k_3 + k_a & -k_3 & 0 & 0 & 0 & 0 & -k_a \\ 0 & -k_3 & k_3 + k_4 & . & 0 & 0 & 0 & 0 \\ 0 & 0 & . & . & . & 0 & 0 & 0 \\ 0 & 0 & 0 & . & . & . & 0 & 0 \\ 0 & 0 & 0 & 0 & . & k_{n-1} & -k_n & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_n & k_n & . \\ -k_a & -k_a & 0 & 0 & 0 & 0 & . & 2k_a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ x_{n-1} \\ x_n \\ x_a \end{bmatrix} = \begin{bmatrix} k_1 y(t) \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[M_1]\{\ddot{x}\}+[C_1]\{\dot{x}\}+[K_1]\{x\}=\{F_1\}$$

The calculations are carried out as following by assuming steady-state

$$\vec{x}(t)=\vec{X}e^{i\omega t}$$

$$\ddot{\vec{x}}(t)=\vec{X}(-\omega^2)e^{i\omega t}$$

$$\{F\}=\{F_0\}e^{i\omega t}=\begin{bmatrix} k_1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \end{bmatrix} e^{i\omega t}$$

$$[M_1]\vec{X}(-\omega^2)+[C_1](i\omega)\vec{X}+[K_1]\vec{X}=\{F_1\}$$

$$(-\omega^2[M_1]+(i\omega)[C_1]+[K_1])\vec{X}=\{F_0\}$$

and the governing equation for the transmissibility is found to be

$$\vec{X}=\frac{\{F_0\}}{(-\omega^2[M_1]+(i\omega)[C_1]+[K_1])}=\frac{b_1}{A_1}$$

The mass value of tuned mass damper, m_a, is selected as 0.3 and the optimum value of k_a and c_a are found to be 12.597 and 1.925 respectively for minimum transmissibility, which is decreased to 2.780. During the optimization process, it is also found that the tuned mass damper is to be connected between m_2 and m_3 to achieve the minimum transmissibility. The corresponding transmissibility graph is plotted in log-log scale and provided in Figure 8 .

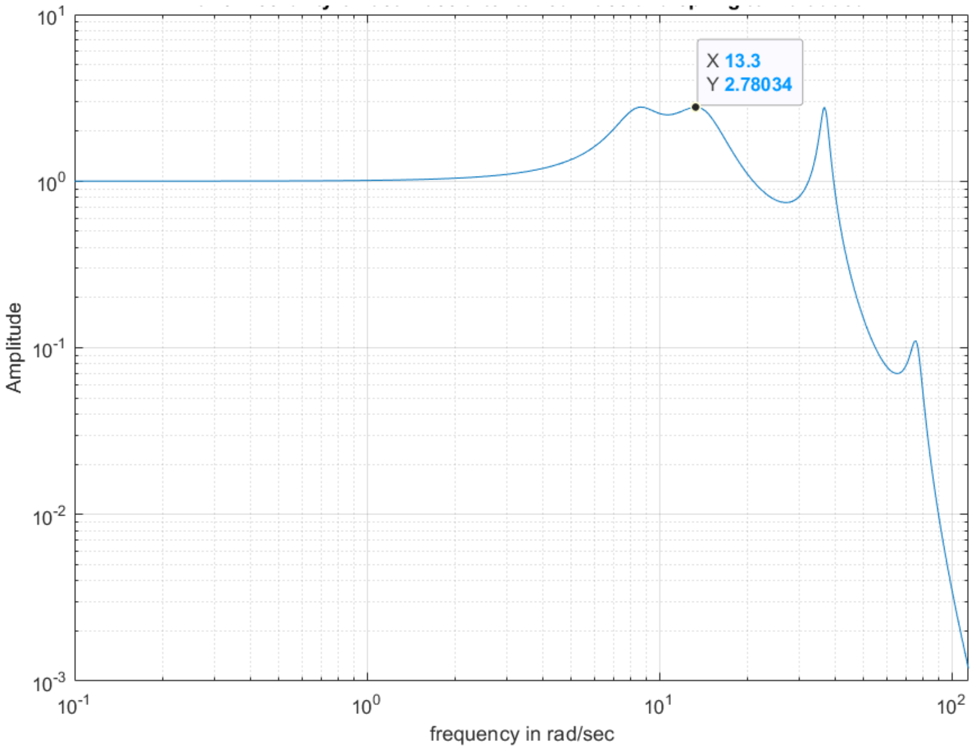

Figure 8: Transmissibility of the System with a Modified Tuned Mass Damper

5. Results and Discussion

With the values that we have used while explaining our calculations *(ma=0.3, n=3*), initial maximum $|\frac{X}{Y}|$ value was *312.698*. After attaching a regular tuned mass damper with optimal values that our MATLAB code found (k_a=25.447 and c_a=2.008), we managed to reduce the maximum $|\frac{X}{Y}|$ value to *2.723*. Later we have modified the regular tuned mass damper system with an additional spring that connected to previous mass. After optimization, we have found k_a=12.597 c_a=1.925 resulting maximum $|\frac{X}{Y}|$ value of *2.780*. Modification of regular tuned mass damper system increased our maximum $|\frac{X}{Y}|$ value. At the tables below, various results of optimized k_a, c_a, optimal locations where tuned mass damper systems should be placed, $|\frac{X}{Y}|$ values of both regular tuned mass damper and modified tuned mass damper and $|\frac{X}{Y}|$ values of system without tuned mass dampers are given which are calculated by our MATLAB code.

Table 2: Calculated Values of Regular Tuned Mass Damper

	k_a	c_a	Location	$\lvert\frac{X}{Y}\rvert$ w/o RTMD	$\lvert\frac{X}{Y}\rvert$ with RTMD	Decrease Rate
n=2, m_a=0.1	11.372	0.442	2	419.813	4.579	91,68
n=2, m_a=0.15	16.980	1.012	1	419.813	5.727	73,30
n=2, m_a=0.2	18.577	1.097	2	419.813	3.315	126,64
n=2, m_a=0.25	21.089	1.443	2	419.813	2.998	140,03
n=2, m_a=0.3	23.028	1.803	2	419.813	2.765	151,83
n=3, m_a=0.1	13.460	0.800	3	312.698	6.136	50,96
n=3, m_a=0.15	17.539	1.163	3	312.698	4.333	72,16
n=3, m_a=0.2	20.398	1.495	3	312.698	3.441	90,87
n=3, m_a=0.25	23.210	1.744	3	312.698	2.970	105,28
n=3, m_a=0.3	25.447	2.008	3	312.698	2.723	114,83
n=4, m_a=0.1	14.968	0.991	4	1244.013	6.966	178,58
n=4, m_a=0.15	19.377	1.441	4	1244.013	4.916	253,05
n=4, m_a=0.2	22.425	1.855	4	1244.013	3.895	319,38
n=4, m_a=0.25	24.471	2.234	4	1244.013	3.286	378,57
n=4, m_a=0.3	28.781	3.037	3	1244.013	3.462	359,33
n=5, m_a=0.1	15.547	1.099	5	170.017	7.259	23,42
n=5, m_a=0.15	20.610	1.585	5	170.017	5.124	33,18
n=5, m_a=0.2	23.540	2.043	5	170.017	4.060	41,87
n=5, m_a=0.25	25.628	2.458	5	170.017	3.425	49,64
n=5, m_a=0.3	27.047	2.835	5	170.017	3.005	56,57

Table 3: Calculated Values of Modified Tuned Mass Damper

	k_a	c_a	Location	$\lvert\frac{X}{Y}\rvert$ w/o MTMD	$\lvert\frac{X}{Y}\rvert$ with MTMD	Decrease Rate
n=2, m_a=0.1	40.767	8.509	1-2	419.813	119.339	3,51
n=2, m_a=0.15	7.672	0.686	1-2	419.813	4.062	103,35
n=2, m_a=0.2	9.254	1.001	1-2	419.813	3.548	118,32
n=2, m_a=0.25	10.519	1.309	1-2	419.813	3.200	131,19
n=2, m_a=0.3	11.519	1.617	1-2	419.813	2.945	142,55
n=3, m_a=0.1	93.292	98.707	2-3	312.698	139.975	2,23
n=3, m_a=0.15	8.607	1.147	2-3	312.698	4.537	68,92
n=3, m_a=0.2	114.062	113.246	2-3	312.698	149.745	2,08
n=3, m_a=0.25	11.407	1.736	2-3	312.698	3.067	101,95
n=3, m_a=0.3	12.597	1.925	2-3	312.698	2.780	112,48
n=4, m_a=0.1	98.613	58.041	1-2	1244.013	154.465	8,05
n=4, m_a=0.15	9.516	1.425	3-4	1244.013	5.012	248,20
n=4, m_a=0.2	11.039	1.830	3-4	1244.013	3.971	313,27
n=4, m_a=0.25	12.086	2.199	3-4	1244.013	3.349	371,45
n=4, m_a=0.3	12.794	2.537	3-4	1244.013	2.937	423,56
n=5, m_a=0.1	7.933	1.164	3-4	170.017	8.389	20,26
n=5, m_a=0.15	180.802	39.341	3-4	170.017	143.705	1,18
n=5, m_a=0.2	11.711	2.019	4-5	170.017	4.101	41,45
n=5, m_a=0.25	12.750	2.427	4-5	170.017	3.460	49,13
n=5, m_a=0.3	13.460	2.797	4-5	170.017	3.035	56,01

Almost all of the values of regular tuned mass damper at table 1 were expected. As m_a increased, k_a, c_a, and decrease rate increased while maximum $|\frac{X}{Y}|$ value decreased. Increase of n value on regular tuned mass damper increased k_a and c_a values. However, decrease rates changed differently. Plural n values had more decrease rate than singular n values. When plural values and singular values compared separately, as n increased, decrease rate of plural values increased while singular values decreased. Overall, obtained values were in order except values of modified tuned mass damper at n=5, m_a=0.15 and n=3 m_a=0.2. When m_a is 0.1 c_a and k_a values are found much larger than following values. Also decrease rate of transmission was much lower at such cases. As m_a increased, optimal k_a and c_a values and decrease rate increases. Increase of k_a and c_a values were not as high as regular tuned mass dampers. Increase of n at modified tuned dampers also increased ka and ca values. Also, decrease rates changed differently. Like regular tuned mass dampers, plural n values had more decrease rate than singular n values. When plural values and singular values compared separately, as n increased, decrease rate of plural values increased while singular values decreased. For natural frequencies and mode shapes, only n value was important.

For n = 2:

ω_{n_1}=11.884 rad/sec

ω_{n_2}= 37.865 rad/sec

$$\begin{bmatrix} 1.0000 & 1.0000 \\ 1.1861 & -1.6861 \end{bmatrix}$$

For n = 3:

ω_{n_1}=12.876 rad/sec

ω_{n_2}= 36.769 rad/sec

ω_{n_3}= 76.041 rad/sec

$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 \\ 1.3904 & -0.5865 & -7.9705 \\ 1.4575 & -0.9393 & 13.1485 \end{bmatrix}$$

For n = 4:

ω_{n_1}= 13.464 rad/sec

ω_{n_2}= 38.596 rad/sec rad/sec

ω_{n_3}= 63.266 rad/sec

ω_{n_4}= 126.738 rad/sec

$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.5325 & -0.0376 & -3.0532 & -17.5250 \\ 1.7208 & -0.6957 & 1.5772 & 139.0228 \\ 1.7526 & -0.8175 & 2.6298 & -229.3148 \end{bmatrix}$$

For n = 5:

ω_{n_1}= 13.850 rad/sec

ω_{n_2}= 40.235 rad/sec

ω_{n_3}= 62.870 rad/sec

ω_{n_4}= 95.012 rad/sec

ω_{n_5}= 190.107 rad/sec

$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.6295 & 0.3610 & -1.7134 & -6.2242 & 30,3 \\ 1.9349 & -0.4299 & -0.1365 & 18.3244 & 530,7 \\ 2.0395 & -0.7716 & 1.0587 & -9.4217 & -4209 \\ 2.0571 & -0.8314 & 1.2843 & -15.7345 & 6944 \end{bmatrix}$$

Each column of the matrices is mode shape of its respective natural frequency. As expected, as *n* increased natural frequencies formed equal number to *n*. Also, mode shape vectors' dimension is increased.

Appendix

```
clc;clear all;close all;
%% PART A
%choose the value of n between 2 and 5
while 'True' % User is asked to enter an n value between 2 and 5
    n = input('Enter n value between 2 through 5 \n');
    if n <= 5 && n>=2 %loop breaks if an appropriate value is entered
        break
    end
end
%mass marix and stiffness matrix
for i=1:n
    M(i,i)=2*(n-i+1)/(n^2+n);
    if i>1 && i<n
        K(i,i)=(50*(n^2+n)/(n-i+1))+(50*(n^2+n)/(n-(i+1)+1));
        K(i,i+1)=-(50*(n^2+n)/(n-(i+1)+1));
        K(i,i-1)=-(50*(n^2+n)/(n-i+1));

    elseif i==1
        K(i,i)=(50*(n^2+n)/(n-i+1))+(50*(n^2+n)/(n-(i+1)+1));
        K(i,i+1)=-(50*(n^2+n)/(n-(i+1)+1));

    else
        K(i,i)=(50*(n^2+n)/(n-i+1));
        K(i,i-1)=-(50*(n^2+n)/(n-i+1));
    end
end
%Finding eigen value and eigen vector
[mode,wn2]=eig(K,M);
figure(1)
hold on
plot([1 4],[0 0],'--')
axis off
for i=1:n
    mode(:,i)=mode(:,i)/mode(1,i);
    fprintf('%1.0f. Natural frequency of the system is:%1.3f
rad/sec\n',i,sqrt(wn2(i,i)))
    fprintf('Mode shape is:\n')
    disp(mode(:,i))
    plot(mode(:,i),'*')
end
a={'Mean position','First Mode','Second Mode','Third Mode','Fourth Mode','Fifth Mode'};
legend(a{1:n+1})
%% PART B
wn=sqrt(wn2(end,end));%largest natural frequency
w=0:0.1:1.5*wn;
b=zeros(n,1);
i=1;
b(1,1)=(50*(n^2+n)/(n-i+1));
for i=1:length(w)
    A=(-w(i)^2.*M)+K;
    X=A\b;
    T(i)=abs(X(end));
end
a_b=max(T);
fprintf('Maximum transmitted |X/Y| at Part B=%1.3f\n',a_b)
figure
loglog(w,T)
grid on
xlabel('frequency in rad/sec')
ylabel('Amplitude')
title('Transmissibility of last mass')

%% PART C
%Mass matrix including the tuning mass ma
while 'True' % User is asked to enter an n value between 2 and 5
    ma= input('Enter the mass value of tuned mass damper between 0.1 and 0.3\n');
    if ma <=0.3 && ma>=0.1 %loop breaks if an appropriate value is entered
        break
    end
```

```
end
x0_c=[100 200];
lb=zeros([length(x0_c) 1]);
up=ones([length(x0_c) 1]);
up=up*1.5*wn;
for i=1:n
optimalPair(i,1:2)=fminimax(@(x)trans(x,ma,M,K,b,n,wn,i),x0_c,[],[],[],[],lb,up,[]);
T(i,1:length(w))=trans(optimalPair(i,1:2),ma,M,K,b,n,wn,i);
Tmax_c(i)=max(T(i,1:length(w))); %Maximum transmissibility value of each location where
tuned mass damper is added
end
b_backup=b;
[a,b]=min(Tmax_c); %Finding minimum of maximum transmissibility value of each location
where tuned mass damper is added
fprintf('Regular tuned mass damper should connect with mass%1.0f\n',b);
fprintf('The optimum value of ka=%1.3f and ca=%1.3f\n',optimalPair(b,1:2))
fprintf('Maximum transmitted |X/Y| at Part C=%1.3f\n',a)
figure
loglog(w,trans(optimalPair(b,1:2),ma,M,K,b_backup,n,wn,b))
grid on
xlabel('frequency in rad/sec')
ylabel('Amplitude')
title('Transmissibility of last mass after tuned mass added')
%% PART D

x0_d=[100 200];
lb=zeros([length(x0_d) 1]);
up=ones([length(x0_d) 1]);
up=up*1.5*wn;


for i=2:n
optimalPair_d(i,1:2)=fminimax(@(x)transd(x,ma,M,K,b_backup,n,wn,i),x0_d,[],[],[],[],lb,u
p,[]);
T(i,1:length(w))=transd(optimalPair_d(i,1:2),ma,M,K,b_backup,n,wn,i);
Tmax_d(i)=max(T(i,1:length(w)));
end

Tmax_d2=Tmax_d(2:n); %since i=1 is not calculated

[a_d,b_d]=min(Tmax_d2);
b_d=b_d+1;            %To adjust position since first mass position was excluded at
min(Tmax)

fprintf('Modified tuned mass damper should connect between mass%1.0f and
mass%1.0f\n',b_d-1,b_d);
fprintf('The optimum value of ka=%1.3f and ca=%1.3f\n',optimalPair_d(b_d,1:2))
fprintf('Maximum transmitted |X/Y| at Part D=%1.3f\n',a_d)
figure
loglog(w,transd(optimalPair_d(b_d,1:2),ma,M,K,b_backup,n,wn,b_d))
grid on
xlabel('frequency in rad/sec')
ylabel('Amplitude')
title('Transmissibility of last mass after tuned mass and spring to ma added')



%function
function TR=trans(x,ma,M,K,b,n,wn,position)
%ka and ca
ka=x(1);
ca=x(2);
M1=zeros(n+1,n+1);
M1(n+1,n+1)=ma;
M1(1:n,1:n)=M;
%Damping and Stiffness matrix
C1=zeros(n+1,n+1);
K1=zeros(n+1,n+1);
K1(1:n,1:n)=K;
for i=1:n+1
    for j=1:n+1
        if i==n+1 && j==n+1
```

```
            C1(i,j)=ca;
            K1(i,j)=K1(i,j)+ka;
        elseif i==n+1 && j==position
            C1(i,j)=-ca;
            K1(i,j)=-ka;
        elseif j==n+1 && i==position
            C1(i,j)=-ca;
            K1(i,j)=-ka;
        elseif i==position && j==position
            C1(i,j)=ca;
            K1(i,j)=K1(i,j)+ka;
        end
    end
end
img=sqrt(-1);
w=0:0.1:1.5*wn;
b1=zeros(n+1,1);
b1(1:n)=b;
for j=1:length(w)
    A1=(-w(j)^2.*M1)+(img*w(j).*C1)+K1;
    X=A1\b1;
    TR(j)=abs(X(n));
end

end

function TR=transd(x,ma,M,K,b,n,wn,position)
%Variable ka and ca
ka=x(1);
ca=x(2);
M1=zeros(n+1,n+1);
M1(n+1,n+1)=ma;
M1(1:n,1:n)=M;
%Damping and Stiffness matrix
C1=zeros(n+1,n+1);
K1=zeros(n+1,n+1);
K1(1:n,1:n)=K;
for i=1:n+1
    for j=1:n+1
        if i==n+1 && j==n+1
            C1(i,j)=C1(i,j)+ca;
            K1(i,j)=K1(i,j)+2*ka;
        elseif i==n+1 && j==position
            C1(i,j)=C1(i,j)-ca;
            K1(i,j)=K1(i,j)-ka;
        elseif j==n+1 && i==position
            C1(i,j)=C1(i,j)-ca;
            K1(i,j)=K1(i,j)-ka;
        elseif i==position && j==position
            C1(i,j)=ca;
            K1(i,j)=K1(i,j)+ka;
        elseif i==position-1 && j==position-1
            K1(i,j)=K1(i,j)+ka;
        elseif j==n+1 && i==position-1
            K1(i,j)=K1(i,j)-ka;
        elseif i==n+1 && j==position-1
            K1(i,j)=K1(i,j)-ka;
        end
    end
end
img=sqrt(-1);
w=0:0.1:1.5*wn;
b1=zeros(n+1,1);
b1(1:n)=b;
for j=1:length(w)
    A1=(-w(j)^2.*M1)+(img*w(j).*C1)+K1;
    X=A1\b1;
    TR(j)=abs(X(n));
end

end
```